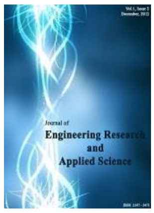
Journal of Engineering Research and Applied Science
Available at www.journaleras.com
Volume 12 (2), December 2023, pp 2301-2306
ISSN 2147-3471 © 2023



Contact Pseudo-Slant Submanifolds of Lorentzian Para Kenmotsu Manifold

Süleyman Dirik[1], Ramazan Sari[2,*]

[1]Department of Mathematics, Faculty of Arts and Sciences, Amasya University, Amasya, Turkey
[2]Department of Mathematics, Faculty of Arts and Sciences, Amasya University, Amasya, Turkey

Accepted 24 July 2023

Abstract
The aim of the present paper is to define and study contact pseudo-slant submanifolds of lorentzian para Kenmotsu manifold. We investigate the geometry of leaves which arise the definition of contact pseudo-slant submanifolds of Lorentzian para Kenmotsu manifold and obtaine integrability conditions of distributions. We also consider parallel conditions of projections on study contact pseudo-slant submanifolds of Lorentzian para Kenmotsu manifold.

Keywords: *Lorentzian para -Kenmotsu manifold, contact pseudo-slant submanifolds*

1. Introduction
Lorentzian Kenmotsu manifolds have been defined by Roşca [16]. Sarı and Turgut Vanlı [17, 18] worked on Lorentzian Kenmotsu manifolds. In [20], Tirpathi and De presented a survey on Lorentzian para-contact manifolds. Also, some authors investigated Lorentzian para Kenmotsu manifolds in [4, 10, 11].

Slant submanifolds are known to generalize invariant and anti-invariant submanifolds, many geometers have expressed an interest in this research. Chen [5, 6] started this research on complex manifolds. Lotta[15] pioneered slant immersions in a almost contact metric manifold. Carriezo defined a new class of submanifolds known as hemi-slant submanifolds (Also known as anti-slant or pseudo-slant submanifolds) [3]. The contact version of a pseudo-slant submanifold in a Sasakian manifold was then defined and studied by V. A. Khan and M. A. Khan. [12]. Later many geometers such as ([7, 13, 14,19]) studied pseudo-slant submanifolds on various manifolds. Recently, M. Atçeken and S. Dirik studied contact pseudo-slant submanifold on various manifolds ([1,8,9]).

In the light of the above studies, our article, the following is how this paper is structured: Section 2 includes some fundamental formulas and definitions of the Lorentzian para-Kenmotsu manifold and it is submanifolds. Section 3 we review some definitions and proves some basic results on the contact pseudo-slant submanifolds of the Lorentzian para-Kenmotsu manifold. Also, the final section looks at the totally umbilical contact pseudo-slant in Lorentzian para Kenmotsu manifolds.

2. Lorentzian Para -Kenmotsu Manifolds
Let $\bar{M}$ be an differentiable manifold with Lorentzian metric g. If we have para-contact structure (φ,ξ,η) on $\bar{M}$ as the following:

$$\varphi^2 X = X + \eta(X)\xi, \quad \eta(\xi) = -1 \tag{1}$$

$$g(\varphi X, \varphi Y) = g(X,Y) + \eta(X)\eta(Y)) \tag{2}$$

$$\varphi(\xi) = 0, \ \ \eta o \varphi = 0, \ \ \eta(X) = g(X,\xi). \tag{3}$$

for all $X, Y \epsilon \Gamma(T\bar{M})$, where φ is (1,1)-tensor field, ξ is a vector field, η is a 1-form, then $\bar{M}$ is a called a Lorentzian almost para-contact metric manifold[20].

From definition, it is clear that $g(\varphi X, Y) = g(X, \varphi Y)$. Similar to any paracontact structure the fundamental 2-form ψ is defined by $\psi(X,Y) = g(\varphi X, Y)$, for all $X, Y \epsilon \Gamma(T\bar{M})$.

Moreover, a almost para contact metric manifold is normal if $[\varphi,\varphi] + 2d\eta \otimes \xi = 0$ where $[\varphi,\varphi]$ is denoting the Nijenhuis tensor field associated to φ. A normal almost para contact metric manifold is called para contact metric manifold.

*Corresponding author: ramazan.sari@amasya.edu.tr

Definition 2.1 Let $\overline{M}$ be an Lorentzian almost para contact metric manifold is said to be an Lorentzian almost para -Kenmotsu manifold if 1-form η are closed (dη=0) and $d\psi = -2\eta \wedge \psi$. A normal almost Lorentzian para -Kenmotsu manifold M is called a Lorentzian para -Kenmotsu manifold.

The following theorem a Lorentzian para contact metric manifold is characterized as LP-Kenmotsu manifold.

Theorem 2.2 Let $(\overline{M}, \varphi, \xi, \eta, g)$ be a Lorentzian para contact metric manifold. $\overline{M}$ is a Lorentzian para Kenmotsu manifold if and only if

$$(\overline{\nabla}_X \varphi)Y = -g(\varphi X, Y)\xi - \eta(Y)\varphi X \tag{4}$$

for all $X, Y \in \Gamma(T\overline{M})$, where $\overline{\nabla}$ denotes the operator of covariant differentiation with respect to the Lorentzian metric g [10].

Corollary 2.3 Let $(\overline{M}, \varphi, \xi, \eta, g)$ a Lorentzian para -Kenmotsu manifold . Then we have

$$\overline{\nabla}_X \xi = -\varphi^2 X \tag{5}$$

for all $X, Y \epsilon \Gamma(TM)$.

3. Submanifolds of Lorentzian Para Kenmotsu Manifold

Let M be a submanifold of a Lorentzian para-Kenmotsu manifold $\overline{M}$. Then Gauss and Weingarten formulas are given by

$$\overline{\nabla}_X Y = \nabla_X Y + \sigma(X, Y) \tag{6}$$

$$\overline{\nabla}_X V = -A_V X + \nabla_X^{\perp} Y \tag{7}$$

for any $X, Y \in \Gamma(TM)$ and $V \in \Gamma(TM)^{\perp}$. σ is the second fundamental from of M, $\nabla^{\perp}$ is the connection in the normal bundle and A_V is the Weingarten endomorphism associated with V. Shape operator A and *t*he second fundamental form σ related by

$$g(\sigma(X, Y), V) = g(A_V X, Y) \tag{8}$$

A submanifold M of $\overline{M}$ is said to be totally geodesic if $\sigma(X, Y) = 0$, for any $X, Y \in \Gamma(TM)$.
On the other hand, the mean curvature tensor H is defined by

$$H = \frac{1}{m}\sum_{k=1}^{m} \sigma(e_k, e_k) \tag{9}$$

where $\{e_1, \dots, e_m\}$ is a local orthonormal basis of TM.

For every tangent vector field X on M we can write

$$\varphi X = PX + FX \tag{10}$$

where PX (resp. FX) denotes the tangential (resp. normal) component of φX. Moreover for every normal vector field V we can state

$$\varphi V = BV + CV \tag{11}$$

where BV (resp. CV) denotes the tangential (resp. normal) component of φV.

For any $X, Y \in \Gamma(TM)$, we have

$$g(\varphi X, Y) = g(X, \varphi Y).$$

From (10), we can see

$$g(PX + FX, Y) = g(X, PY + FY)$$

So we obtain

$$g(PX, Y) = g(X, PY).$$

On the other hand, for any $X \in \Gamma(TM)$ and $V \in \Gamma(TM)^{\perp}$ we have

$$g(\varphi X, V) = g(X, \varphi V).$$

From (10) and (11) , we write

$$g(PX + FX, V) = g(X, BV + CV).$$

Thus ve have

$$g(FX, V) = g(X, BV).$$

Finally,for any $W, V \in \Gamma(TM)^{\perp}$ we can state

$$g(\varphi W, V) = g(W, \varphi V)$$

From (11), we write

$$g(BW + CW, V) = g(W, BV + CV).$$

So we obtain

$$g(CW, V) = g(W, CV).$$

We can summarize all these results by the following proposition.

Proposition 3.1 Let $(\overline{M}, \varphi, \xi, \eta, g)$ be a Lorentzian para contact metric manifold. Then we have

$$g(PX,Y) = g(X,PY),$$

$$g(CW,V) = g(W,CV),$$
$$g(FX,V) = g(X,BV).$$

For any $X, Y \in \Gamma(TM)$ and for $W, V \in \Gamma(TM)^{\perp}$. Suppose that $\xi \in \Gamma(TM)$. Then we have

$$\varphi\xi = P\xi + F\xi = 0.$$

Since $\overline{TM} = TM \oplus TM^{\perp}$, it is obvius that

$$P\xi = F\xi = 0.$$

On the other hand ,we have

$$\eta(\varphi X) = g(\varphi X, \xi) = g(PX + FX, \xi) = g(PX, \xi) + g(FX, \xi) = \eta(PX) + \eta(FX) = 0.$$

And thus we get $\eta oP = \eta oF = 0$.

After following similar steps we have

$$\begin{aligned} \varphi^2 X = \varphi(PX + FX) &= \varphi(PX) + \varphi(FX) \\ &= P^2X + FPX + BFX + CFX \\ &= X + \eta(X)\xi \end{aligned}$$

Since $P^2X + BFX \in \Gamma(TM)$ and $FPX + CFX \in \Gamma(TM)^{\perp}$ we get

$$P^2 + BF = I + \eta \otimes \xi \quad \text{and } FP + CF = 0.$$

Similar to

$$\begin{aligned} \varphi^2 V = \varphi(BV + CV) &= \varphi(BV) + \varphi(CV) \\ &= C^2V + BCV + PBV + FBV \\ &= V, \end{aligned}$$

since $C^2 + FB \in \Gamma(TM)^{\perp}$ and $BC + PB \in \Gamma(TM)$ we get

$$C^2 + FB = I \quad \text{and } BC + PB = 0.$$

We can summarize all these results as following.

Proposition 3.2 . Let $(\overline{M}, \varphi, \xi, \eta, g)$ be a Lorentzian para contact metric manifold. Then we have

$$P\xi = F\xi = 0 \text{ and } \eta oP = \eta oF = 0,$$

$$P^2 + BF = I + \eta \otimes \xi \quad \text{and } FP + CF = 0,$$

$$C^2 + FB = I \quad \text{and } BC + PB = 0.$$

Also defined are the covariant derivatives of the tensor fields P, F, B, and C.

$$(\nabla_X P)Y = \nabla_X PY - P\nabla_X Y \qquad (12)$$
$$(\nabla_X F)Y = \nabla_X^{\perp} FY - F\nabla_X Y \qquad (13)$$
$$(\nabla_X B)V = \nabla_X BV - B\nabla_X^{\perp} V \qquad (14)$$
$$(\nabla_X C)V = \nabla_X^{\perp} CV - C\nabla_X^{\perp} V \qquad (15)$$

the covariant derivative of φ can be defined by

$$(\overline{\nabla}_X \varphi)Y = \overline{\nabla}_X \varphi Y - \varphi\overline{\nabla}_X Y \qquad (16)$$

for any $X, Y \in \Gamma(TM)$ and $V \in \Gamma(T^{\perp}M)$. Where $\tilde{\nabla}$ is the Riemannian connection on $\overline{M}$.

Now, for later use, we establish a result for a submanifold Lorentzian para Kenmotsu manifold.

Proposition 3.3 Let M be submanifold of Lorentzian para Kenmotsu manifold $\overline{M}$. Then we have

$$(\nabla_X P)Y = A_{FY}X + B\sigma(X,Y) - g(PX,Y)\xi - \eta(Y)PX \qquad (17)$$

$$(\nabla_X F)Y = C\sigma(X,Y) - h(X,PY) - \eta(Y)FX \qquad (18)$$

$$(\nabla_X B)V = A_{CV}X - PA_V X - g(FX,V)\xi \qquad (19)$$

$$(\nabla_X C)V = -\sigma(BV,X) - FA_V X \qquad (20)$$

for all $X, Y \in \Gamma(TM)$ and $V \in \Gamma(T^{\perp}M)$.

Proposition 3.4. Let M be submanifold of Lorentzian para Kenmotsu manifold $\overline{M}$. Then we have the following results:

P is parallel if and only if $A_{FY}X = A_{FX}Y$, for all $X, Y \epsilon \Gamma(TM)$.
F is parallel if and only if $A_V PY = A_{CV}Y$, for all $Y \epsilon \Gamma(TM)$ and $V \in \Gamma(T^{\perp}M)$.
B is parallel if and only if $A_{CV}X = PA_V X$, for all $X \epsilon \Gamma(TM)$ and $V \in \Gamma(T^{\perp}M)$.
C is parallel if and only if $A_V BU = -A_U BV$, for all $V, U \in \Gamma(T^{\perp}M)$.
Using (5), (6), (7), and (8), we have that ξ is tangent to M.

$$\nabla_X \xi = X + \eta(X)\xi \qquad (21)$$

$$\sigma(X, \xi) = 0 \qquad (22)$$

for all$X \in \Gamma(TM)$.

Let us now same definitions of classes submanifolds. If F is identically zero in (10), then the submanifold is invariant.

If P is identically zero in (10), then the submanifold is anti-invariant,
If there is a constant angle $\theta(x) \in \left[0, \frac{\pi}{2}\right]$ between φX and TM for all nonzero vector X tangent to M at x, the manifold is called slant.
A proper slant submanifold is one that is not invariant or anti-invariant. i. e. As a result, the following theorem characterized slant submanifolds of almost contact metric manifolds;

Theorem 3.5: [2]. Let M be a slant submanifolds of an almost contact metric manifold $\bar{M}$ such that $\xi \in \Gamma(TM)$, then, M is a slant if and only if a constant $\lambda \in [0, 1]$ exists such that

$$P^2 = \lambda(\mathrm{I} + \eta \otimes \xi) \tag{23}$$

furthermore, in this situation, if θ is the slant angle of M. Then it satisfies $\lambda = \cos^2\theta$.

Corollary3.6: [2]. Let M be a slant submanifolds of an almost contact metric manifold $\bar{M}$. Then for all $X, Y \in \Gamma(TM)$ we have

$$g(PX, PY) = \cos^2\theta\{g(X,Y) + \eta(X)\eta(Y)\} \tag{24}$$

$$g(FX, FY) = \sin^2\theta\{g(X,Y) + \eta(X)\eta(Y)\}. \tag{25}$$

4. Contact Pseudo-Slant Submanifolds of Lorentzin Para Kenmotsu Manifold

In this section, in a Lorentzian para Kenmotsu manifold, necessary and sufficient conditions are given for a submanifold to be a contact pseudo-slant submanifold.

Let M be a slant submanifold of an almost Lorentzian paracontact metric manifold $\bar{M}$. M is said to be pseudo-slant of $\bar{M}$ if there exit two orthogonal distributions D_θ and $D^\perp$ on M such that:

TM has the orthogonal direct decomposition

$$TM = D^\perp \oplus D_\theta, \xi \in D_\theta$$

The distribution D_θ is slant with slant angle, that is, the slant angle between of D_θ and φD_θ is a constant.The distribution $D^\perp$ is an anti-invariant, That is,

$$\varphi D^\perp \subset T^\perp M \text{ [12]}.$$

Let d_1 and d_2 be dimensional of distributions $D^\perp$ and D_θ respectively. Then

If $d_2 = 0$ then, M is an anti-invariant submanifold.
If $d_1 = 0$ and $\theta = 0$ then, M is an invariant submanifold.
If $d_1 = 0$ and $\theta \in \left(0, \frac{\pi}{2}\right)$ then, M proper slant submanifold.
If $\theta = \frac{\pi}{2}$ then, M is an anti-invariant submanifold.
If $d_1\, d_2 \neq 0$ and $\theta \in \left(0, \frac{\pi}{2}\right)$ then, M is a proper pseudo-slant submanifold.
If $d_1\, d_2 \neq 0$ and $\theta = 0$ then, M is a semi-invariant submanifold.

From the definitions, we can see that a slant submanifold is a generalization of invariant (if $\theta = 0$)and anti-invariant $\left(\text{if } \theta = \frac{\pi}{2}\right)$ submanifolds.
If the orthogonal complementary of φTM in $T^\perp M$ is denoted by V, then the normal bundle $T^\perp M$ can be decombosed as follows.

$$T^\perp M = FD_\theta \oplus FD^\perp \oplus \nu, \quad FD_\theta \perp FD^\perp.$$

Definition 4.1 A contact pseudo slant submanifold M of a Lorentzian para-Kenmotsu manifold $\bar{M}$ is said to be mixed-geodesic submanifold if
$\sigma(X,Y) = 0$ for all $X \in \Gamma(D_\theta)$, $Y \in \Gamma(D^\perp)$.

Theorem 4.2. Let M be proper contact pseudo slant submanifold of a Lorentzian para-Kenmotsu manifold $\bar{M}$. M is either an anti-invariant or a mixed geodesic if B is parallel.

Proof: For all $X \in \Gamma(D_\theta)$, $Y \in \Gamma(D^\perp)$, from (18) and (19)
B parallel if and only if F parallel, thus $\nabla F = 0$.
This implies

$$C\sigma(X,Y) - \sigma(X,PY) - \eta(Y)FX = 0.$$

Replacing X by PX in the above equation, we get

$$C\sigma(PX,Y) - \sigma(PX,PY) = 0$$

for $Y \in \Gamma(D^\perp)$, $PY = 0$. Hence

$$C\sigma(PX,Y) = 0.$$

Replacing X by PX in the above equation, we have

$$C\sigma(P^2X,Y) = C\cos^2\theta\sigma(X,Y) = 0.$$

Hence we have either $\sigma(X,Y) = 0$ (M is mixed geodesic) or $\theta = \frac{\pi}{2}$ (M is anti-invariant).

Theorem 4.3. Let M be totally umbilical proper contact pseudo slant submanidold of a Lorentzian para-Kenmotsu manifold $\bar{M}$. If B is parallel, then M is either minimal or anti-invariant submanifold.

Proof: For all $X \in \Gamma(D_\theta)$, $Y \in \Gamma(D^\perp)$, from (18) and (19), we have:
B parallel if and only if F parallel, so $\nabla F = 0$.
This implies

$$C\sigma(X,Y) - \sigma(X,PY) - \eta(Y)FX = 0.$$

Replacing X by PX in the above equation, we get

$$C\sigma(PX,Y) - \sigma(PX,PY) = 0$$

for $Y \in \Gamma(D^\perp)$, $PY = 0$. Hence

$$C\sigma(PX,Y) = 0.$$

Since M is totally umbilical, from (12)

$$Cg(PX,Y)H = 0$$

replacing X by PX in the above equation, we have

$$Cg(P^2X,Y)H = Cg(PX,PY)H = C\cos^2\theta g(X,Y)H = 0.$$

Hence we have either $\theta = \frac{\pi}{2}$ (M is anti-invariant) or $H = 0$ (M is minimal).

Theorem 4.4. Let M be a contact pseudo slant submanifold of a Lorentzian para-Kenmotsu manifold $\bar{M}$. Then $D^\perp$ is integrable at all times.

Proof: For all $W, U \in \Gamma(D^\perp)$, from (4), we have

$$(\bar{\nabla}_W\varphi)U = -g(\varphi W,U)\xi - \eta(U)\varphi W = 0.$$

By using (6), (7) , (10) and (11) we have

$$-A_{FU}W + \nabla^\perp_W U - P\nabla_W U - F\nabla_W U - B\sigma(W,U) - C\sigma(W,U) = 0.$$

Comparing the tangent companents, we have

$$-A_{FU}W - P\nabla_W U - B\sigma(W,U) = 0 \qquad (26)$$

interchanging W and U , we get

$$-A_{FW}U - P\nabla_U W - B\sigma(U,W) = 0\,. \qquad (27)$$

Subtracting equation (26) from (27) and using the fact that σ is symmetric , we get

$$A_{FU}W - A_{FW}U + P[W,U] = 0,$$

$$P[U,W] = A_{FU}W - A_{FW}U. \qquad (28)$$

On the other hand, for all $Z \in \Gamma(TM)$. By using (6), (7) (8) and (16), we have

$$\begin{aligned} g(A_{FU}W - A_{FW}U, Z) &= g(\sigma(Z,W),FU) - g(\sigma(U,Z),FW) \\ &= g(\sigma(Z,W),FU) - g(\tilde{\nabla}_Z U, FW) \\ &= g(\sigma(Z,W),FU) + g(\varphi\tilde{\nabla}_Z U, W) \\ &= g(\sigma(Z,W),FU) + g(-A_{FU}Z + \nabla^\perp_Z FU, W) \\ &= g(\sigma(Z,W),FU) - g(A_{FU}Z, W) \\ &= g(\sigma(Z,W),FU) + g(\sigma(Z,W),FU) = 0 \end{aligned}$$

Here

$$A_{FU}W = A_{FW}U.$$

So, from (28), $[U,W] \in \Gamma(D^\perp)$, for all $W, U \in \Gamma(D^\perp)$. That is, $D^\perp$ is every time integrable.

Theorem 4.5. Let M be a contact pseudo slant submanifold of a Lorentzian para-Kenmotsu manifold $\bar{M}$. Then the D_θ is integrable if and only if

$$\varpi_1\{\nabla_X PY - A_{FY}X - P\nabla_Y X - B\sigma(X,Y) + \eta(Y)PX\} = 0.$$

for all $X, Y \in \Gamma(D_\theta)$.
Proof: Let ϖ_1 and ϖ_2 the projections on $D^\perp$ and D_θ , respectively. For all $X, Y \in \Gamma(D_\theta)$ from (4), we have

$$(\bar{\nabla}_X\varphi)Y = -g(\varphi X,Y)\xi - \eta(Y)\varphi X.$$

On applying (6), (7), (10) and (11), we get

$$\nabla_X PY + \sigma(X,PY) - A_{FY}X + \nabla^\perp_X FY - P\nabla_X Y - F\nabla_X Y - B\sigma(X,Y) - C\sigma(X,Y) + g(\varphi X,Y)\xi + \eta(Y)\varphi X = 0.$$

Comparing the tangential components

$$\nabla_X PY - A_{FY}X - P\nabla_X Y - B\sigma(X,Y) + g(\varphi X,Y) + \eta(Y)PX = 0,$$

$$\nabla_X PY - A_{FY}X - P\nabla_Y X + P\nabla_Y X - P\nabla_X Y - B\sigma(X,Y) + g(\varphi X,Y)\xi + \eta(Y)PX = 0\,,$$

$$P[X,Y] = \nabla_X PY - A_{FY}X - P\nabla_Y X - B\sigma(X,Y) + g(\varphi X,Y)\xi + \eta(Y)PX \qquad (29)$$

$X, Y \in \Gamma(D_\theta), [X,Y] \in \Gamma(D_\theta), so\ \varpi_1 P[X,Y] = 0.$
As a result, we conclude our theorem by applying ϖ_1 to both sides of (29) equation.

Theorem 4.6. Let M be a totally umbilical contact pseudo slant submanifold of a Lorentzian para-Kenmotsu manifold $\bar{M}$. Then at least one of the following satements is true.

i)M is proper contact pseudo slant submanifold,
ii) $H \in \Gamma(\nu)$,
iii) $Dim\,(D^\perp) = 1$.
Proof: Let $X \in \Gamma(D^\perp)$ and using (4), we obtain

$$(\bar{\nabla}_X\varphi)X = -g(\varphi X,X)\xi - \eta(X)\varphi X = 0.$$

On applying (6), (7), (10) and (11), we get

$$-\mathrm{A_{FX}X} + \nabla_X^{\perp}\mathrm{FX} - \mathrm{F}\nabla_{\mathrm{X}}\mathrm{X} - \mathrm{B}\sigma(X,X) - \mathrm{C}\sigma(X,X) = 0.$$

Comparing the tangential components

$$\mathrm{A_{FX}X} + \mathrm{B}\sigma(X,X) = 0.$$

Taking the product by $\mathrm{Z} \in \Gamma(\mathrm{D}^{\perp})$,we obtain

$$\mathrm{g}(\mathrm{A_{FX}X},\mathrm{Z}) + \mathrm{g}(\mathrm{B}\sigma(X,X),Z) = 0.$$

Because M is a totally umbilical , we get

$$\begin{aligned} 0 &= \mathrm{g}(\mathrm{A_{FX}Z},\mathrm{X}) + \mathrm{g}(\mathrm{B}\sigma(X,X),Z) \\ &= \mathrm{g}(\sigma(Z,X),FX) + \mathrm{g}(\sigma(X,X),FZ) \\ &= \mathrm{g}(Z,X)\mathrm{g}(H,FX) + \mathrm{g}(X,X)\mathrm{g}(H,FZ) \\ &= \mathrm{g}(X,X)\mathrm{g}(BH,Z) + \mathrm{g}(Z,X)\mathrm{g}(BH,X) \end{aligned}$$

that is

$$\mathrm{g}(BH,Z)\mathrm{X} + \mathrm{g}(BH,X)Z = 0.$$

Here BH is either zero or X and Z are linearly dependent vector fields. If $BH \neq 0$, than dim $(\mathrm{D}^{\perp}) = 1$. Otherwise $H \in \Gamma(\mu)$. Since $\mathrm{D}_\theta \neq 0$ M is contact pseudo slant submanifold. Since $\theta \neq 0$ and $\mathrm{d}_1\mathrm{d}_2 \neq 0$ proper contact pseudo slant submanifold.

References

[1] M. Atçeken and S.Dirik , On the geometry of pseudo-slant submanifold of a Kenmotsu manifold, Gulf Journal of Mathematics , 2(2)(2014),51-66.

[2] J. L. Cabrerizo, A. Carriazo, L. M. Fernandez, M. Fernandez, Slant submanifolds in Sasakian manifolds, Glasgow Mathematical Journal, 42, (2000), 125-138.

[3] A. Carriazo, New Devolopments in Slant Submanifolds Theory, Narosa publishing House, New Delhi, India, (2002).

[4] V. Chandra ,S. Lal , On 3-dimensional Lorentzian para Kenmotsu manifolds, Dif. Geo. Dyn. Sys., 22, 87-94, (2020)

[5] B. Y. Chen, Geometry of slant submanifolds, Katholieke Universiteit Leuven, Leuven, (1990).

[6 B.Y. Chen, Slant immersions, Bulletin of the Australian Mathematical Society, 41, (1990), 135-147.

[7] U.C. De and A. Sarkar, On Pseudo-slant submanifolds of trans-Sasakian manifolds, Proceedings of the Estonian Academy of Sciences, 60, 1(2011), 1-11, doi: 10.3176/proc.2011.1.01.

[8] S. Dirik, M. Atçeken, Pseudo-slant submanifolds in Cosymplectic space forms, Acta Universitatis Sapientiae: Mathematica, 8, 1(2016), 53-74, doi: 10.1515/ausm-2016-0004.

[9] S. Dirik, M. Atçeken, ¨U . Yildirim, Pseudo-slant submanifold in Kenmotsu space forms, Journal of Advances in Mathematics, 11, 10(2016), 5680-5696.

[10] H. Haseeb , R. Prasad , Certain results on Lorentzian para Kenmotsu manifolds, Bol. Soc. Paran. Mat., 39(3), 201-220, (2021)

[11] H. Haseeb , R. Prasad , Some resultson Lorentzian Kenmotsu manifolds, Bulletin of the Transilvania University of Brasov. Mathematics, Informatics, Physics. Series II, 13(1), 185-198, (2020).

[12] V. A. Khan and M. A. Khan, Pseudo-slant submanifolds of a Sasakian manifold, Indian Journal of püre and applied Mathematics, 38, 1(2007), 31-42.

[13] M. A. Khan, Totally umbilical Hemi-slant submanifolds of Cosymplectic manifolds, Mathematica Aeterna, 3, 8(2013), 645-653.

[14] B. Laha and A. Bhattacharyya, Totally umbilical Hemislant submanifolds of LP-Sasakian Manifold, Lobachevskii Journal of Mathematics, 36, 2(2015), 127-131, doi: 10.1134/S1995080215020122.

[15] A. Lotta, Slant submanifolds in contact geometry, Bulletin of Mathematical Society Romania, 39, (1996), 183-198.

[16] R. Roşca , On Lorentzian Kenmotsu manifolds , Atti Accad. Peloritana Pericolanti, Cl. Sci., 69, 15-30, (1991).

[17] R. Sarı, Some properties curvture of Lorentzian Kenmotsu manifolds, Applied Mathematics and Nonlinear Sciences, 5(1), 283-292, (2020).

[18] R. Sarı ,A.T. Vanli , Slant submanifolds of Lorentzian Kenmotsu manifold, Mediterranean Journal of Mathematics, 16, 129, (2019).

[19] İ. Ünal, Generic submanifolds of Lorentzian para Kenmotsu manifold, KMU Journal of Engineering and Natural Sciences, 3(2), (2021), 79-85.

[20] M. M Tripathi ,U.C. De ., Lorentzian almost paracontact manifolds and their submanifolds, The Pure and Applied Mathematics, 8.2, 101-125, (2001).